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Collapse Examination of WTC 7

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Symbols:

a = acceleration, [m/s^2]
F = total force, [N]
F_g = force of the Earth's gravity [N]
F_n = average resistance force of the structure
F_v = reacting force caused by flowing masses, [N]
g = the acceleration of the Earth's gravity, 9.807 m/s^2
h = the lowering of the top of the tower, [m]
h_0 = the original height of the tower, 174 m
I = momentum [kgs]
$\dot{m}$ = flow of masses, [kg/s]
t = time, [s]
q = the characteristic mass of the tower, [kg/m]
v = velocity, [m/s]

WTC building no. 7 collapsed on 9/11/2001 at 5.20 p.m., approximately 7 hours after WTC towers 1 and 2 had collapsed. The collapse fracture began at the bottom, and the building appeared to sink into the ground as an intact whole.

Let's examine the collapse process with the help of figure 1. The original height of the tower is h_0 and the lowering of the top of the building is h. The mass of the undamaged part of the tower can be obtained from:

$$m = q(h_0 - h) \qquad (1)$$

where q is the characteristic mass of the tower. For simplicity, let's assume that the mass of the tower is distributed uniformly. The velocity of the undamaged part of the tower is v.

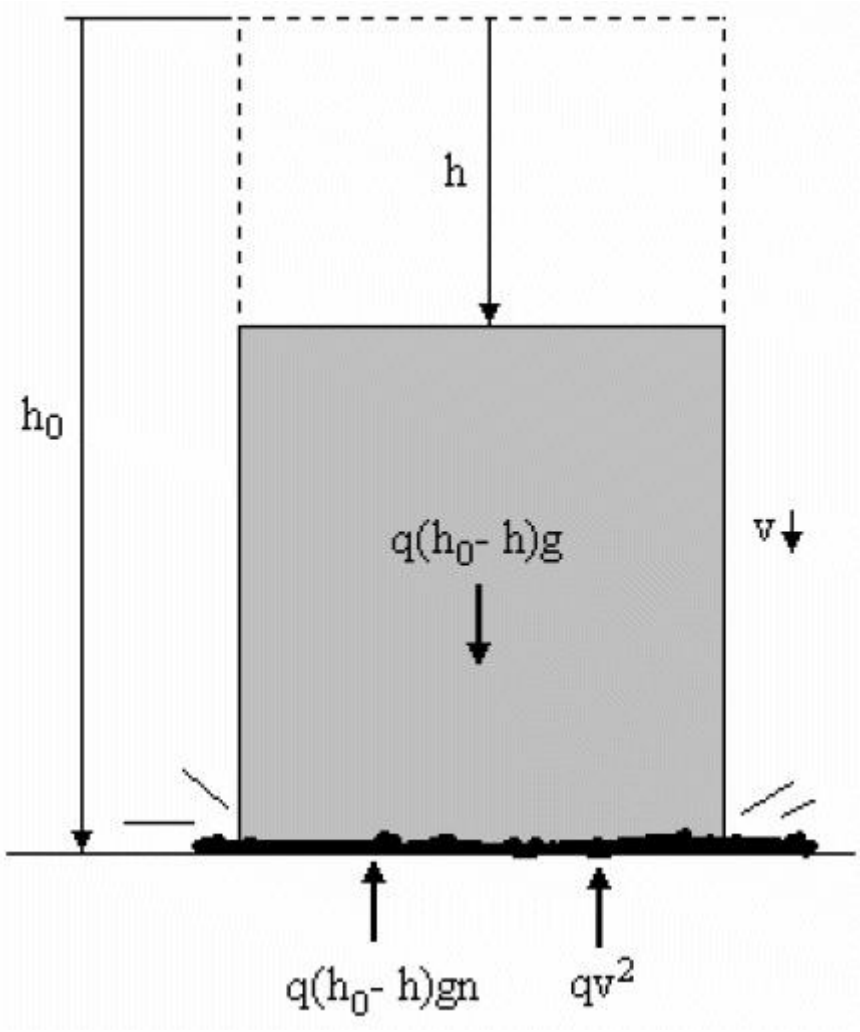

Figure 1: The collapse process.

The momentum of the undamaged part of the tower can be obtained from this equation:

$$I = q(h_0 - h)v \qquad (2)$$

The differentially small change in momentum ***dI*** can be obtained from this equation:

$$dI = q(h_0 - h)dv - qvdh \qquad (3)$$

The forces affecting the undamaged part are the Earth's gravity F_g, the average resistance force F_n and reacting force F_v caused by the flowing masses. The Earth's gravity can be obtained from the equation:

$$F_g = q(h_0 - h)g \qquad (4)$$

where ***g*** is the acceleration of the Earth's gravity. The average resistance force of the structure can be obtained from the equation:

$$F_n = -q(h_0 - h)gn \qquad (5)$$

where ***n*** is the average resistance factor of the structure. Normally $n = 1$, with the result that the tower remains stable and does not collapse. The actual resistance is much higher, since the building was designed to withstand earthquakes and tornados. The minus sign before the equation indicates that the force is directed upwards.

A force directed downward is caused by the flowing masses of the building. The force equals the flow of masses $\dot{m}$ x velocity $\boldsymbol{v}$ (here = qv^2). This causes a reacting force F_v, which can be obtained from:

$$F_v = -qv^2 \tag{6}$$

The total force $\boldsymbol{F}$ affecting the undamaged part of the building can be obtained from last three equations by summing them:

$$F = q(h_0 - h)(1-n)g - qv^2 \tag{7}$$

According to basic dynamics, *force x time = change in momentum*. Here time is understood as the differentially short time $\boldsymbol{dt}$:

$$Fdt = dI \tag{8}$$

Let's substitute equations (3) and (7) into equation (8):

$$q(h_0 - h)(1-n)gdt - qv^2dt = q(h_0 - h)dv - qvdh \tag{9}$$

Simplified and divided by dt:

$$(h_0 - h)(1-n)g - v^2 = (h_0 - h)\frac{dv}{dt} - v\frac{dh}{dt} \tag{10}$$

From basic dynamics, we obtain acceleration ***a***:

$$a = \frac{dv}{dt} \tag{11}$$

and velocity ***v*** from:

$$v = \frac{dh}{dt} \tag{12}$$

Substituting equations (11) and (12) into equation (10) we obtain:

$$(h_0 - h)(1-n)g - v^2 = (h_0 - h)a - v^2 \tag{13}$$

Simplifying, we obtain acceleration *a*:

$$a = (1-n)g \tag{14}$$

Acceleration is constant, which significantly facilitates the rest of the examination. From basic dynamics, we obtain the time ***t*** elapsed during the collapse with the help of $\boldsymbol{h_0}$, the original height of the building:

$$t = \sqrt{\frac{2h_0}{a}} = \sqrt{\frac{2h_0}{(1-n)g}} \qquad (15)$$

The results are as follows:

The height of WTC 7: 174 m
The observed collapse time: 6.5 s
The fall of an apple from the top of the tower: 7.0 s
The fall of an apple from the top of the tower in a vacuum: 6.0 s
The resistance factor during the observed collapse time: 0.16

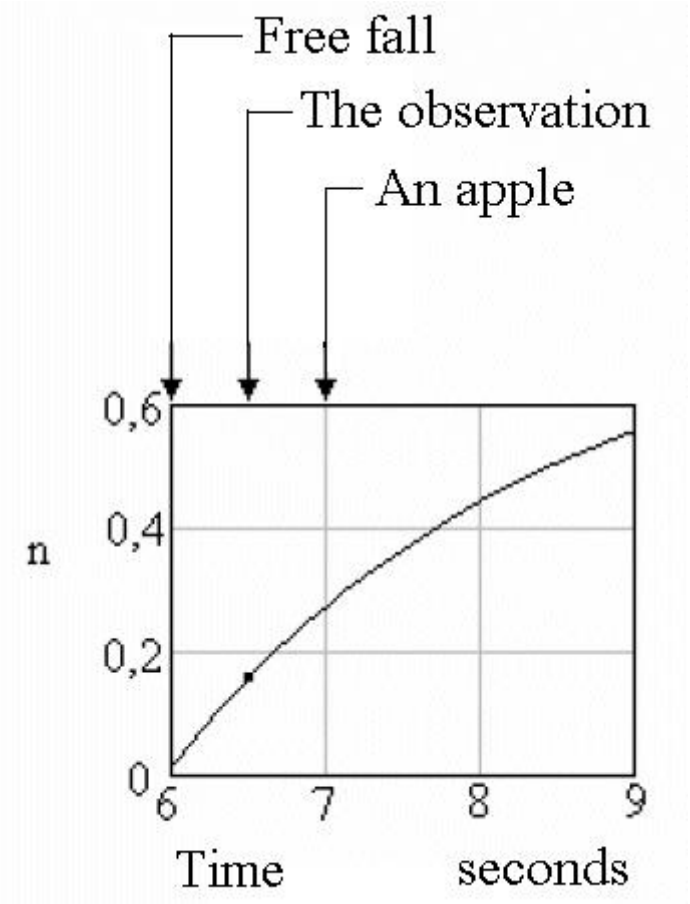

Figure 2: The results represented graphically

Conclusions

The observed collapse time of WTC 7 was 6.5 seconds. That is only half a second longer than it would have taken for the top of the building to fall to the ground in a vacuum, and half a second shorter than the falling time of an apple when air resistance is taken into account. The apple is 6 cm in diameter and weighs 100 g (thereby fulfilling the EU requirements).

With the observed collapse time we obtain the resistance factor n = 0.16 by using equation (15).

The great speed of the collapse and the low value of the resistance factor strongly suggest controlled demolition.